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THE COMPONENT REALIZABILITY OF CUBIC DECOMPOSITIONS OF ORDER 10, THAT HAVE ORDER TYPE 1021

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Продовжено дослідження кубічних розкладів порядку 10. Для типу 1021розрізнено реалізовні та нереалізовні компонентні типи. Для кожного реалізовного компонентного типу побудовано реалізуючий розклад.

More than ten years ago there was done the partition of the set of cubic decompositions of the complete graph of order 10 into 14 types [1].Under the type of the cubic decomposition R of the graph K_n we understand the vector $a(R)=(a_4, a_6,...)$, where a_i means the number of components of order i in the decomposition R. The type is an invariant with regard to isomorphism in the set of cubic decompositions of order n. Such types we call the *order* types.

The above mentioned order types for the decompositions of K_{10} are
1.0003 2.0130 3.0211 4.0500 5.1021 6.1102 7.1310
8.2120 9.2201 10.3011 11.3300 12.4110 13.5001 14.6100.

It is proved in [2] that all these order types except for 6100 are realizable. The next step in the solution of the existence problem of the cubic decomposition is the introduction of *component* types. For the decomposition R of order type 1021, the component type has the ‘chemical’ formula G_iXYK_4. Here G_i means the canonical form of the eldest component in the decomposition R, and X, Y are the canonical forms of components of order 8. The component types of the other order types are defined in the similar way.

We take the canonical graphs G_i (i=1,…, 19) from [3], and the graphs G_{20}, G_{21} have the edge lists

G_{20}: 12 13 14 23 24 34 56 57 58 67 69 7A 89 8A 9A,

G_{21}: 12 13 14 23 24 34 56 59 5A 68 69 6A 78 79 7A.

Further we denote A–E the connected cubic graphs of order 8 as it is shown in Fig.1. We write F instead of $2K_4$.

It is easy to count that the set of cubic decompositions of order 10 with order type 1021 is divided into $21\cdot(6+C_7^2)=567$ different component types. The *deep* existence problem of cubic decompositions of order 10 is that to indicate those component types for which the corresponding decomposition sets are nonempty. The component types that have such a quality we call *realizable*.

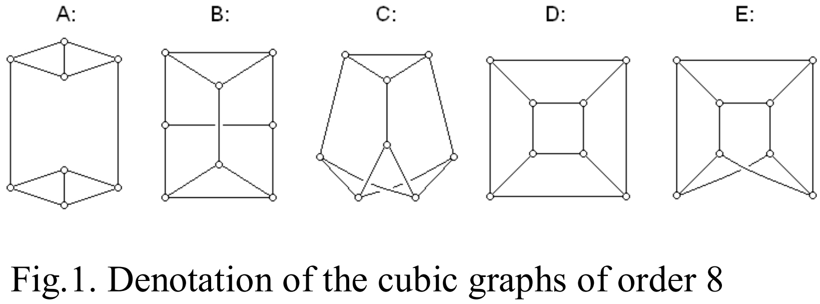

Fig.1. Denotation of the cubic graphs of order 8

The authors have compiled the computer program to compose the cubic decompositions of a given component type. With the help of the program they have obtained the solution for the above formulated problem for order type 1021. The result is in the next theorem.

Theorem. *From all the component types for order type 1021 only these are realizable:*

G_1ABK_4 G_1ACK_4 G_1ADK_4 G_1BBK_4 G_1BDK_4 G_1BEK_4 G_1CDK_4;
G_2AAK_4 G_2ABK_4 G_2BBK_4 G_2CCK_4 G_2DDK_4;
G_3ABK_4 G_3ACK_4 G_3BBK_4 G_3BCK_4 G_3BDK_4 G_3BEK_4 G_3CCK_4 G_3CDK_4 G_3DEK_4 G_3EEK_4;
G_4ABK_4 G_4BBK_4 G_4BCK_4 G_4BDK_4 G_4BEK_4 G_4DEK_4;
G_5ACK_4 G_5AEK_4 G_5BBK_4 G_5BCK_4 G_5BDK_4 G_5BEK_4 G_5CCK_4 G_5CDK_4 G_5CEK_4;
G_6AAK_4 G_6ABK_4 G_6ACK_4 G_6ADK_4 G_6AEK_4 G_6BBK_4 G_6BCK_4 G_6BDK_4 G_6BEK_4 G_6BFK_4 G_6CCK_4 G_6CDK_4 G_6CEK_4 G_6EEK_4;
G_7ABK_4 G_7ACK_4 G_7AEK_4 G_7BBK4 G_7BCK_4 G_7BDK_4 G_7BEK_4 G_7CCK_4 G_7CDK_4 G_7CEK_4 G_7DDK_4 G_7DEK_4;
G_8ABK_4 G_8ACK_4 G_8ADK_4 G_8AEK_4 G_8BBK_4 G_8BCK_4 G_8BDK_4 G_8BEK_4 G_8CCK_4 G_8CEK_4 G_8CFK_4;
G_9AAK_4 G_9ABK_4 G_9BBK_4 G_9BCK_4 G_9BEK_4 G_9CDK_4 G_9CEK_4 G_9EEK_4;
$G_{10}ABK_4$ $G_{10}BBK_4$ $G_{10}BCK_4$ $G_{10}BEK_4$ $G_{10}CCK_4$;
$G_{11}ABK_4$ $G_{11}ACK_4$ $G_{11}AEK_4$ $G_{11}BBK_4$ $G_{11}BCK_4$ $G_{11}BDK_4$ $G_{11}BEK_4$ $G_{11}CDK_4$ $G_{11}CEK_4$ $G_{11}DEK_4$;
$G_{12}ACK_4$ $G_{12}BBK_4$ $G_{12}BCK_4$ $G_{12}BEK_4$ $G_{12}CEK_4$;
$G_{13}AAK_4$ $G_{13}ABK_4$ $G_{13}ACK_4$ $G_{13}BBK_4$ $G_{13}BCK_4$ $G_{13}BDK_4$ $G_{13}BEK_4$ $G_{13}CCK_4$ $G_{13}CDK_4$ $G_{13}CEK_4$ $G_{13}ACK_4$ $G_{13}EEK_4$;
$G_{14}AAK_4$ $G_{14}ABK_4$ $G_{14}ACK_4$ $G_{14}ADK_4$ $G_{14}BBK_4$ $G_{14}BCK_4$ $G_{14}BEK_4$ $G_{14}CCK_4$ $G_{14}CEK_4$ $G_{14}DFK_4$;
$G_{15}ABK_4$ $G_{15}ACK_4$ $G_{15}ADK_4$ $G_{15}AEK_4$ $G_{15}BBK_4$ $G_{15}BCK_4$ $G_{15}BDK_4$ $G_{15}BEK_4$ $G_{15}EFK_4$;
$G_{16}AAK_4$ $G_{16}ABK_4$ $G_{16}ACK_4$ $G_{16}ADK_4$ $G_{16}AEK_4$ $G_{16}BBK_4$ $G_{16}BCK_4$ $G_{16}BDK_4$ $G_{16}BEK_4$ $G_{16}BFK_4$ $G_{16}CCK_4$ $G_{16}CDK_4$;
$G_{17}ABK_4$ $G_{17}AEK_4$ $G_{17}BBK_4$ $G_{17}BCK_4$ $G_{17}BDK_4$ $G_{17}BEK_4$ $G_{17}CCK_4$ $G_{17}CEK_4$ $G_{17}EEK_4$;
$G_{18}AAK_4$ $G_{18}ABK_4$ $G_{18}ADK_4$ $G_{18}AFK_4$ $G_{18}BBK_4$ $G_{18}BCK_4$ $G_{18}BEK_4$ $G_{18}CCK_4$ $G_{18}CEK_4$ $G_{18}DEK_4$;
$G_{19}AAK_4$ $G_{19}BBK_4$;
$G_{21}CDK_4$.

It should be remarked, that there are no cubic decomposition with the eldest component isomorphic to G_{20}.

To confirm the theorem we give the realization for each realizable component type.

The eldest component is G_1 = 1–234 2–34 3–5 4–5 5–6 6–78 7–9X 8–9X 9–X

G_1ABK_4 : 1–579 2–67X 3–69X 5–79 6–X; 1–68X 2–589 4–69X 5–8X 6–9;[3478];
G_1ACK_4 : 1–59X 2–789 3–78X 5–9X 7–8; 1– 678 2–56X 4–78X 5–78 6–X;[2569];
G_1ADK_4 : 1–579 2–57X 3–69X 5–7 6–9X; 1–68X 2–689 4–69X 5–89X;[3478];
G_1BBK_4 : 1–567 2–689 3–789 5–78 6–9; 1–89X 2–57X 4–789 5–9X 7–8;[346X];
G_1BDK_4 : 1–579 3–47X 4–67 5–9X 6–9X; 1–68X 2–69X 3–689 4–89A;[2578];
G_1BEK_4 : 1–79X 2–69X 3–467 4–79 6–X; 1–568 3–89X 4–68X 5–9X 6–9;[2578];
G_1CDK_4 : 1–567 3–479 4–7X 5–9X 6–9X ; 1–89X 2–69X 3–68X 4–689;[2578];

The eldest component is G_2 = 1–234 2–34 3–5 4–6 5–67 6–8 7–9X 8–9X 9–X

G_2AAK_4 : 1–589 2–58X 3–69X 5–8 6–9X; 1– 67X 2–679 4–59X 5–9X 6–7;[3478];
G_2ABK_4 : 1–589 2–68X 3–69X 5–89 6–X; 1–67X 2–579 4–59X 5–X 6–79;[3478];
G_2BBK_4 : 1–568 2–89X 3–69X 5–89 6–X; 1–79X 2–567 4–59X 5–X 6–79;[3478];
G_2CCK_4 : 1–568 2–89X 4–59X 5–8 6–9X; 1–79X 2–567 3–69X 5–9X 6–7;[3478];
G_2DDK_4 : 1–569 2–56X 3–69X 4–59X; 1–78X 2–678 5–89X 6–79X;[3478];

The eldest component G_3= 1–234 2–34 3–5 4–6 5–78 6–79 7–X 8–9X 9–X

G_3ABK_4 : 1–789 2–79X 3–48X 4–8X 7–9;2–568 3–679 4–579 5–9 6–8 7–8; [156X];
G_3ACK_4 : 1–59X 2–689 3–68X 5–9X 6–8; 1–678 2–57X 4–58X 5–6 6–X 7–8; [3479];
G_3BBK_4 : 1–567 2–78X 4–58X 5–6 6–X 7–8; 1–89X 2–569 3–68X 5–9X 6–8; [3479];
G_3BCK_4 : 1–567 2–79X 4–59X 5–6 6–X 7–9; 1–89X 2–568 3–69X 5–9X 6–8; [3478];
G_3BDK_4 : 1–569 3–468 4–57 5–9 6–8 7–89; 1–78X 2–7893–79X 4–89X;[256X];
G_3BEK_4 : 1–789 2–689 3–467 4–79 6–8; 2–57X 3–89X 4–58X 5–9 7–89;[156X];
G_3CCK_4 : 1–679 2–789 3–468 4–79 6–8; 1–58X 3–79X 4–58X 5–9 7–89;[256X];
G_3CDK_4 : 1–569 3–467 4–58 5–9 6–8 7–89;1–78X 2–789 3–89X 4–79X;[256X];
G_3DEK_4 : 1–589 2–89X 4–59X 5–6 6–8X; 1–67X 2–567 3–69X 5–9X 7–9;[3478];
G_3EEK_4 : 1–578 2–789 3–489 4–57 5–9; 1–69X 3–67X 4–89X 6–8 7–89;[256X];

The eldest component is G_4= 1–234 2–34 3–5 4–6 5–78 6–9X 9–X 7–89 8–X

G_4ABK_4 : 1–569 2–79X 3–67X 5–69 7–X; 1–78X 2–568 4–57X 5–X 6–78;[3489];
G_4BBK_4 : 1–579 2–69X 3–67X 5–69 7–X; 1–68X 2–578 4–57X 5–X 6–78;[3489];
G_4BCK_4 : 1–578 2–58X 3–67X 5–X 6–78; 1–69X 2–679 4–57X 5–69 7–X;[3489];
G_4BDK_4 :1–569 2–679 3–67X 5–9X 7–X;1–78X 2–58X 4–57X 5–6 6–8 7–X; [3489];
G_4BEK_4 : 1–578 2–568 3–67X 5–X 6–8 7–X; 1–69X 2–79X 4–57X 5–69 6–7; [3489];
G_4DEK_4 : 1–567 2–56X 3–67X 4–57X ; 1–89X 2–789 5–69X 6–78 7–X;[3489];

The eldest component is G_5= 1–234 2–34 3–5 4–6 5–78 6–9X 7–9X 8–9X

G_5ACK_4 : 1–569 2–789 3–678 5–69 7–8; 1–78X 2–56X 4–578 5–X 6–78;[349X];
G_5AEK_4 : 1–57X 2–59X 3–479 4–79 5–X; 1–689 3–68X 4–58X 5–69 9–X;[2678];
G_5BBK_4 : 1–579 2–689 3–678 5–69 7–8; 1–68X 2–57X 4–578 5–X 6–78;[349X];
G_5BCK_4 : 1–578 2–568 3–467 4–78 5–6; 1–69X 2–79X 3–89X 6–78 7–8;[459X];

G_5BDK_4 : 1–57X 2–59X 3–479 4–7X 5–9; 1–689 3–68X 4–89X 5–69X;[2678];
G_5BEK_4 : 1–57X 2–59X 3–479 –4–7X 5–9; 1–689 3–68X 4– 589 5–6X 9–X;[2678];
G_5CCK_4 : 1–689 2–689 3–479 4–78 6–7; 1–57X 2–57X 3–68X 5–6 6–8 7–8;[459X];
G_5CDK_4 : 1–579 2–57X 3–49X 4–57 9–X; 1–68X 2–689 4–89X 5–69X;[3678];
G_5CEK_4: : 1–578 3–789 4–78X 5–9X 9–X; 1–69X 2–59X 3–46X 4–59 5–6;[2678];

The eldest component is G_6 : 1–234 2–35 3–6 4–57 5– 8 6–79 7–X 8–9X 9–X

G_6AAK_4 : 1–789 2–49X 3–48X 4–X 7–8 7–9; 2–678 3–579 4–689 5–79 6–8;[156X]; G_6ABK_4 : 1–567 3–489 4–89 5–67 6–8 7–9; 1–89X 2–789 3–57X 5–9X 7–8;[246X];
G_6ACK_4 : 1–567 3–489 4–89 5–67 6–8 7–9; 1–89X 2–789 3–57X 5–9X 7–8;[246X];
G_6ADK_4 : 1–789 2–48X 3–79X 4–8X 7–9;2–679 3–458 4–69 5–79 6–8 7–8;[156X]; G_6AEK_4 : 2–48X 3–59X 4–8X 5–79 7–89; 1–789 2–678 3–478 4–69 6–8;[156X];
G_6BBK_4 : 1–568 2–46X 3–48X 4–8 5–6X; 1–79X 2–789 4–69X 6–8X 7–8;[3579];
G_6BCK_4 :1–567 2–79X 3–59X 5–6 6–X 7–9;1–89X 3–478 4–9X 5–79X 7–8;[2468];
G_6BDK_4 : 2–469 3–458 4–6 5–79 6–8 7–89; 1–789 2–78X 3–79X 4–89X;[156X];
G_6BEK_4 : 1–567 2–789 3–589 5–6 6–8 7–9; 1–89X 3–47X 4–89 5–79X 7–8;[246X];
G_6BFK_4 : 1–789 2–79X 3–48X 4–9X 7–8; 2–468 4–68 6–8 3–579 5–79 7–9;[156X];
G_6CCK_4 : 1–567 2–789 3–789 5–69 6–8; 1–89X 3–45X 4–89 5–7X 7–89;[246X];
G_6CDK_4 : 1–569 3–457 4–89 5–6 6–8 7–89;1–78X 2–789 3–89X 5–79X;[246X];
G_6CEK_4 :1–567 2–79X 3–79X 5–69 6–X; 1–89X 3–458 4–9X 5–7X 7–89;[2468];:
G_6EEK_4 : 1–578 2–789 3–457 4–89 5–9; 1–69X 3–89X 5–67X 6–8 7–89;[246X];

The eldest component is G_7 : 1–234 2–35 3–6 4–57 5–8 6–9X 7–89 8–X 9–X

G_7ABK_4 : 1–567 3–49X 4–9X 5–69 6–7 7–X; 1–89X 2–79X 3–578 5–7X 8–9;[2468];
G_7ACK_4 : 1–569 3–47X 4–9X 5–69 6–7 7–X; 1–78X 2–79X 3–589 5–7X 8–9;[2468];
G_7AEK_4 : 1–569 2–468 3–489 4–8 5–69; 1–78X 2–79X 4–69X 6–78 8–9;[357X];
G_7BBK_4 : 1–56X 2–49X 4–8X 5–69 6–8 8–9; 1–789 2–467 3–489 4–69 6–7;[357X];
G_7BCK_4 : 1–569 2–67X 3–79X 5–9X 6–7; 1–78X 3–458 4–6X 5–67 6–8 7–X;[2489];
G_7BDK_4 : 1–578 2–79X 3–89X 5–7X 8–9; 1–69X 3–457 4–9X 5–69 6–7 7–X;[2468];
G_7BEK_4 : 1–568 2–469 3–489 4–8 5–69; 1–79X 2–78X 4–69X 6–78 8–9;[357X];
G_7CCK_4 : 1–569 2–79X 3–79X 5–6X 6–7; 1–78X 3–458 4–9X 5–79 7–X 8–9;[2468];

G_7CDK_4 : 1–569 3–479 4–6X 5–9X 6–7 7–X; 1–78X 2–67X 3–58X 5–67 6–8;[2489];
G_7CEK_4 : 1–569 3–479 4–6X 5–9X 6–7 7–X; 1–78X 2–67X 3–58X 5–67 6–8;[2489];
G_7DDK_4 : 1–578 3–458 4–6X 5–X 6–78 7–X; 1–69X 2–67X 3–79X 5–679;[2489];
G_7DEK_4 : 1–679 2–67X 3–79X 5–69X; 1–58X 3–458 4–6X 5–7 6–78 7–X;[2489];

The eldest component is G_8 : 1–234 2–35 36 4–57 58 6–9X 7–9X 8–9X

G_8ABK_4 : 1–569 2–678 3–789 5–69 7–8; 1–789 3–45X 4–68 5–7X 6–78 ;[249X];
G_8ACK_4 : 1–567 2–468 3–478 4–8 5–67; 1–89X 2–79X 4–69X 6–78 7–8;[359X];
G_8ADK_4 : 1–59X 2–48X 3–458 4–8 5–9 9–X; 2–679 3–79X 4–69X 5–67X ;[1678];
G_8AEK_4 : 1–579 2–49X 3–47X 4–X 5–79; 1–68X 3–589 4–689 5–6X 6–8;[2678];
G_8BBK_4 : 1–569 2 79X 3–57X 5–6 6–7 9–X; 1–78X 3–489 4–9X 5–79X 7-8;[359X];
G_8BCK_4 : 1–568 2–467 3–478 4–8 5–67; 1–79X 2–89X 4–69X 6–78 7–8;[3459X];
G_8BDK_4 : 1–569 2–49X 3–45X 4–6 5–6 9–X; 1–78X 3–789 4–89X 5–79X;[2678];
G_8BCK_4 : 1–569 2–479 4–89 5–67 6–8 7–8; 1–78X 2–68X 3–478 4–6X 6–7;[359X];
G_8CCK_4 : 1–568 2–478 3–478 4–6 5–67; 1–79X 2–69X 4–89X 6–78 7–8;359X];
G_8CEK_4 : 1–59X 2–47X 3–47X 4–9 5–79; 2–689 3–89X 4–68X 5–6 9–X;[1678]:
G_8CFK_4 : 1–89X 2–79X 3–478 4–9X 7 8; 2–468 4–68 6–8 3–59X 5–9X 9–X;[1567];

The eldest component is G_9: 12 13 14 23 25 36 47 48 57 58 69 6A 79 8A 9A

G_9ABK_4 : 1–569 2–678 3–789 5–69 7–8; 1–78X 2–49X 4–69 6–78 7–X 8–9;[345X];
G_9BBK_4 : 1–567 2–689 3–789 5–69 7–8; 1–89X 2–47X 4–69 6–78 7–X 8–9;[345X];
G_9BCK_4 : 1–567 2–68X 3–78X 5–6X 7–8; 1–89X 2–479 4–6X 6–78 7–X8–9;[3459]; G_9BEK_4 : 1–567 2–478 4–69 5–69 7–8 8–9; 1–89X 2–69X 3–789 6–78 7–X;[345X];
G_9CDK_4 : 1–69X 2 69X 3–789 6–8 7–8X; 1–578 2–478 4–69 5–69 6–7 8–9;[3459];
G_9DEK_4 : 1–59X 2–48X 3–48X 4–5 5–9 8–9; 2–679 3–579 4–69X 5–6X7–X;[1678];
G_9EEK_4 : 1–59X 2–479 3–579 4–5X 7–X; 2–68X 3–48X 4–69 5 69X 8–9;[1678];

The eldest component is G_{10} : 1–234 2–35 3–6 4–78 5–79 6–7X 8–9X 9–X

$G_{10}ABK_4$: 1–568 2–679 3–589 5–68 7–9; 1–79X 2–48X 4–69 6–89 7–8X;[345X];
$G_{10}BBK_4$: 1–567 2–78X 3–58X 5–6 6–8 7–X;1–89X 3–479 4–5X 5–8X 7–89;[2469];
$G_{10}BCK_4$: 1–2–467 3–59X 4–56 5–X 6–9 7–9X;1–79X 2–89X 3–478 4–9X 7–8; [1589];
$G_{10}BEK_4$: 1–79X 2–459 3–57X 4–59 5–X; 2–68X 3–489 4–6X 6–9 7–89X;[1568];
$G_{10}CCK_4$: 1–567 2–789 3–789 5–68 6–9; 1–89X 2–46X 4–69 6–8 7–89X;[345X];

The eldest component is G_{11} : 1–234 2–35 3–6 4–78 5– 79 6– 89 7–X 8–X 9–X

$G_{11}ABK_4$: 1–568 2–479 4–69 5–68 7–89; 1–79X 2–68X 3–789 6–7X 8–9;[345X];

$G_{11}ACK_4$: 1–56X 3–89X 5–6X 6–7 7–89 8–9;1–789 2–789 3–457 4–59 5–8;[246X];
$G_{11}AEK_4$: 1–568 2–46X 3–48X 4–X 5–68; 1–79X 3–579 4–569 5–X 6–7X;[2789];
$G_{11}BBK_4$: 1–569 2–789 3–578 5–X 5–6 6–7 8–9; 1–789 3–49X 4–59 5–8X;[246X];
$G_{11}BCK_4$: 1–567 2 689 3–789 5–68 7–9; 1–89X 2–47X 4–69 6–7X 7–8 8–9;[345X]; $G_{11}BDK_4$: 1–568 2–46X 3–48X 4–6 5–8X; 1–79X 3–579 4–59X 5–6 6–7X;[2789];
$G_{11}BEK_4$:1–56X 2–47X 3–457 4–X 5–6 6–7;2–689 3–89X 4–569 5–8X 6–X;[1789];
$G_{11}CDK_4$: 1–68X 2–68X 4–59X 5–8X 6–X; 1–57X 2–47X 3–45X 56 6–7;[3789];
$G_{11}CEK_4$:1–56X 2–47X 3–457 4–6 5–X 6–7;2–689 3–89X 4–59X 5–68 6–X;[1789];
$G_{11}DDK_4$: 1–57X 2–79X 3–59X 5–8 8–9; 1–79X 3–579 4–569 5–X 6–7X;[456X];

The eldest component is G_{12} : 1–234 2–35 3–6 4–78 5–9X 6–79 7–8 8–X 9–X
$G_{12}ACK_4$:1–568 2–789 3–579 5–6 6–8 7–9;1–79X 3–48X 4–59 5–78 7–X 8–9;[246X];
$G_{12}BBK_4$:1–567 2–789 3–589 5–6 6–8 7–9;1–89X 3–47X 4–59 5–78 7–X 8–9;[246X];
$G_{12}BCK_4$:1–679 3–478 4–59 5–68 6–87–9;1–58X 2-789 3–59X 5–7 7–X 8–9;[246X];
$G_{12}BEK_4$:1–58X 3–47X 4–59 5–8 7–9X 8–9;1–679 2–789 3–589 5–67 6–8;[246X];
$G_{12}CEK_4$:1–567 3–478 4–59 5–6 6–8 7–9 8–9;1–89X 2–789 3–59X 5–78 7–X;[246X];

The eldest component is G_{13} : 1–234 2–35 3–6 4–78 5–79 6–8X 7–X 8–9 9–X
$G_{13}AAK_4$:1–679 2–46X 3–469 4–X 6–7 7–9;2–789 3–578 4–569 5–6 6–9 7–8;[158X];
$G_{13}ABK_4$: 1–568 2–679 3–789 5–68 7–9;1–79X 2–48X 4–69 6–79 7–8 8–X;[345X];
$G_{13}ACK_4$: 1–568 2–479 4–69 5–68 7–89; 1–79X 2–68X 3–789 6–79 8–X;[345X];
$G_{13}BBK_4$: 1–567 2–469 3–479 4–5 5–6 7–9; 1–89X 2–78X 4–69X 6–79 7–8;[358X];
$G_{13}BCK_4$:1–58X 2–478 3–47X 4–5 5–X 7–8;2–69X 3–589 4–69X 5–68 8–X;[1679];
$G_{13}BDK_4$: 1–679 2–468 3–489 4–6 7–89; 2–79X 3–57X 4–59X 5–6 6–79; [158X];
$G_{13}BEK_4$:1–567 3–478 4–6X 5–6X 7–8 8–X;1–89X 2–48X 3–59X 4–59 5–8;[2679];
$G_{13}CCK_4$: 1–578 2–48X 3–47X 4–5 5–X 8–X; 1–69X 3–589 4–69X 5–68 8–X;[2679];
$G_{13}CDK_4$: 1–567 2–479 3–479 4–5 5–6 6–9; 1–89X 2–68X 4–69X 6–7 7–89;[358X];
$G_{13}CEK_4$: 1–58X 2–47X 3–47X 4–5 5–8 7–8; 2–689 3–589 4–69X 5–6X 8–X;[1679];

$G_{13}DDK_4$: 1–579 3–79X 4–59X 5–8 7–8 8–X; 1–68X 2–48X 3–458 4–6 5–6X;[2679];

$G_{13}EEK_4$:1–57X 2–48X 3–47X 4–5 5–8 7–8;1–689 3–589 4–69X 5–6X 8–X;[2679];

The eldest component is G_{14} : 1–234 2–56 3–56 4–78 5–9 6–X 7–9X 8–9X

$G_{14}AAK_4$: 1–57X 2–49X 4–69 5–7X 6–79;1–689 3–49X 4–5X 5–68 6–8 9–X;[2378];

$G_{14}ABK_4$: 1–567 2–49X 4–5X 5–7 6–79 9–X; 1–89X 3–49X 4–69 5–68X 6–8;[2378];

$G_{14}ACK_4$: 1–69X 2–78X 5–678 6–9 7–8 9–X;1–578 3–78X 4–56X 5–X 6–78;[2349];

$G_{14}ADK_4$: 1–56X 2–38X 3–78 5–6X 6–7 7–8; 1–789 2–479 4–56 5–78 6–89;[349X];

$G_{14}BBK_4$: 1–579 2–478 4–69 5–78 6–89; 1–68X 3–478 4–5X 6–7 7–8;[239X]

$G_{14}BCK_4$: 1–57X 2–478 4–6X 5–6X 6–8 7–8;1–689 3–478 4–59 5–78 6–79;[239X]

$G_{14}BEK_4$: 1–57X 2–789 5–6X 6–89 7–89–X;1–689 3–789 4–569 5–78 6–7;[234X]

$G_{14}CCK_4$: 1–569 2–49X 3–49X 4–6 5–6X; 1 78X 4–59X 5–78 6–789 9–X;[2378]

$G_{14}CEK_4$: 1–67X 2–347 3–79 4–6X 9–X; 1–589 2–89X 3–48X 4–59 5–X;[5678]

$G_{14}DFK_4$: 1–79X 2–49X 4–56 5–7X 6–79; 1568 3–49X 4–9X 5–68 6–8 9–X;[2378]

The eldest component is G_{15} : 1–234 2–56–3–57 4–68 5–9–6–X 7–89 8–X 9–X

$G_{15}ABK_4$: 1–568 2–489 3–469 4–9 5–68; 1–79X 2–38X 3–8X 6–789 8–9;[457X]

$G_{15}ACK_4$: 1–568 2–348 3–49 4–9 5–68 6–9; 1–79X 2–79X 3–68X 6–78 8–9;[457X];

$G_{15}ADK_4$: 1–568 2–349 3–46 4–9 5–68 8–9; 1–79X 2–78X 3–89X 6–789;[457X];

$G_{15}AFK_4$: 1–567 2–47X 3–46X 4–X 5–67; 1–89X 4–579 5–8X 6–789 7–X;[2389];

$G_{15}BBK_4$: 1–569 2–489 3–468 4–9 5–68; 1–78X 2–37X 3–9X 6–789 8–9;[457X]

$G_{15}BCK_4$: 1–569 2–49X 4–79 5–6X 6–7 7–X;1–78X 2–378 3–4X 4–5X 5–78;[3689];

$G_{15}BDK_4$: 1–567 2–47X 3–46X 4–5 5–6 –7–X; 1–89X 4–79X 5–78X 6–789;[2389]

$G_{15}BEK_4$: 1–567 2–47X 3–46X 4–7 5–6X; 1–89X 4–59X 5–6X 6–79 7–X;[2389]

$G_{15}EFK_4$: 1–57X 2–78X 3–68X 5–68 6–7; 1–689 4–57X 5–7X 6–89 7–X 8–9;[2349]

The eldest component is G_{16} : 1–234 2–56 3–57 4–68 5–9 6–X 7–8X 8–9 9–X

$G_{16}AAK_4$: 1–57X 3–469 4–9X 5–7X 6–79; 1–689 2–479 4–57 5– 68 6–8 7–9;[238X]

$G_{16}ABK_4$: 1–567 2–479 3–469 4–9–5–67; 1–89X 4–57X 5–8X 6–789 7–9; [238X]

$G_{16}ACK_4$: 1–56X 2–479 4–7X 5–6X 6–9 7–9; 1–789 3–469 4–59 5–78 6–78;[238X]

$G_{16}ADK_4$: 1–58X 4–79X 5–8X 6–789 7–9; 1–679 2–479 3–469 4–5 5–67;[238X]

$G_{16}AEK_4$: 1–57X 3–689 5–7X 6–89 7–9 8–X; 1–689 2–789 4–579 5–68 6–7;[234X]

$G_{16}BBK_4$: 1–567 2–789 3–689 5–68 7–9; 1–89X 4–579 5–7X 6–789 8–X;[234X]

$G_{16}BCK_4$: 1–57X 2–789 5–6X 6–89 7–9 8–X; 1–689 3–689 4–579 5–78 6–7;[234X]

$G_{16}BDK_4$: 1–567 2–479 3–469 4–5 5–6 7–9; 1–89X 4–79X 5–78X 6–789;[238X]

$G_{16}BEK_4$: 1–789 3–68X 5–67X 6–9 7–9 8–X; 1–56X 2–78X 4–57X 5–8 6–78;[2349]
$G_{16}BFK_4$: 1–79X 3–469 4–5X 5–7X 6–79; 1–568 2–479 4–79 5–68 6–8 7–9;[238X]
$G_{16}CCK_4$: 1–589 3–469 4–57 5–8 6–78 7–9; 1–67X 2–479 4–9X 5–67X 6–9;[238X]
$G_{16}CDK_4$: 2–478 3–89X 4–59 5–7X 7–9 8–X; 1 79X 2–39X 3–46 4–7X 6–79;[1568]

The eldest component is G_{17} : 1–234 2–56 3–57 4–68 5–9 6–X 7–9X 8–9X

$G_{17}ABK_4$: 1–56X 2–378 3–8X 5–6X 6–7 7–8; 1–789 3–469 4–57 5–78 6–89;[249X]
$G_{17}AEK_4$: 1–56X 2–478 4–7X 5–6X 6–8 7–8; 1–789 3–468 4–59 5–78 6–79;[239X]
$G_{17}BBK_4$: 1–567 2–478 3–468 4–7 5–68; 1–89X 4–59X 5–7X 6–789 7–8;[239X]
$G_{17}BCK_4$: 1–567 2–78X 3–68X 5–7X 6–8; 1–89X 4–57X 5–68 6–79 7–8 9–X;[2349]
$G_{17}BDK_4$: 1–567 2–478 3–468 4–5 5–6 7–8; 1–89X 4–79X 5–78X 6–789;[239X]
$G_{17}BEK_4$: 1–567 2–478 3–468 4–5 5–7 6–8; 1–89X 4–79X 5–68X 6–79 7–8; [239X]
$G_{17}CCK_4$: 1–56X 2–37X 3–68 5–8X 6–7 7–8; 1–789 2–489 4–57 5–67 6–89; [349X]
$G_{17}CEK_4$: 1–568 2–378 3–46 4 57 5–8 6–7; 1–79X 3–89X 5–67X 6–89 7–8; [249X]
$G_{17}EEK_4$: 1–579 2–489 4–57 5–6 6–89 7–8; 1–68X 2–37X 3–68 5–78X 6–7;[349X]

The eldest component is G_{18} :1–234 2–56 3–57 4–89 5–8 6–9X 7–9X 8–X

$G_{18}AAK_4$:1–567 2–47X 3–46X 4–X 5–67;1–89X 4–567 5–9X 6–78 7–8 9–X; [2389]
$G_{18}ABK_4$:1–569 2–478 4–67 5–69 7–8; 1–78X 3–468 4–5X 5–7X 6–78; [239X]
$G_{18}ADK_4$:1–59X 4–67X 5–9X 6–78 7–8 8–9; 1–678 2–478 3–468 4–5 5–67; [239X]
$G_{18}AFK_4$:1–678 2–47X 3–46X 4–X 6–8 7–8;2–389 3–89 4–567 5–67 6–7 8–9; [159X]
$G_{18}BBK_4$: 1–567 2–478 3–468 4–5 5–6 7–8; 1–89X 4–67X 5–79X 6–78 8–9; [239X]
$G_{18}BCK_4$:1–569 3–469 4–67 5–79 7–8 8–9; 1–78X 2–478 4–5X 5–X 6–578;[239X]
$G_{18}BEK_4$:1–59X 2–48X 3–489 4–5 5–X 8–9; 2–379 3–6X 4–67X 5–679 9–X; [1678]
$G_{18}CCK_4$:1–59X 2–48X 3–48X 4–5 5–9 8–9; 2–379 3–69 4–67X 5–67X 9–X; [1678]
$G_{18}CEK_4$:1–569 3–468 4–57 5–9 6–7 7–8 8–9; 1–78X 2–478 4–6X 5–67X 6–8; [239X]
$G_{18}DEK_4$:1–69X 4–67X 5–79X 6–8 7–8 8–9; 1–578 2–478 3–468 45 5–6 6–7; [239X]

The eldest component is G_{19} : 1–234 2–56 3–78 4–9X 5–7X 6–89 7–9 8–X

$G_{19}AAK_4$: 1–568 2–39X 3–6X 5–68 8–9 9–X; 1–79X 3–459 4–56 5–9 6–7X 7–X; [2478]
$G_{19}BBK_4$: 1–568 2–789 5–89 6–7X 7–X 9–X; 1–79X 2–34X 3–9X 4–78 7–8 8–9; [3456]

The eldest component is G_{21} : 1–234 2–34 3–4 5–89X 6–89X 7–89X

$G_{21}CDK_4$:1–589 2–689 3–789 5–67 6–7; 1–67X 2–57X 3–56X 4–567; [489X]

Conclusions and perspectives

The order types 0003 and 0050 were completely investigated in [4, 5]. In [6] we had recently solved the enumeration problem for order types 3011, 4110, 5001. The deep existence problem in the cases of order types 1102 and 0211 is solved in [7, 8].

The possible continuation of the work is the investigation of the cubic decompositions of the graphs K_{13}, K_{16} etc. We can point out the papers [8, 9] which have begun to elaborate the direction. The other direction in developing the topic is to investigate the decompositions of complete graphs into regular graphs of degree $k>3$. Finally, one may examine cubic decompositions of arbitrary regular graphs.

REFERENCES

1. A.J. Petrenjuk, On the enumeration of cubic decompositions of the complete graph K(10) (Ukrainian). The Fifth International conference named after Academician M. Kravtchuk, Report Theses, Kyiv, 1996, p.332.
2. L.P. Petrenjuk, A.J. Petrenjuk, The realizability of types of cubic decompositions of the graph K_{10} (Ukrainian). The Third International Science and Practical Conference 'Mathematical and Programming Maintenance of Intellectual Systems (MPZIS–2005)'. November 16–18, 2005, Report Theses, Dnipropetrovs'k, 2005, 139–140.
3. A.M.Barajev, I.A.Faradjev, The construction and the computer investigation of regular and regular bipartite graphs (Russian). In: 'Algorithmic Investigations in Combinatorics', Moscow, 1978.
4. A.J.Petrenjuk, Enumerating decompositions of K(10) into isomorphic cubic factors. In: 'Svitogl'ad', vyp.2, Kirovograd, 1996, 52–60.
5. L.P. Petrenjuk, A.J. Petrenjuk, On the enumeration of non-isomorphic decompositions of the graph K(10) into cubic factors (Russian), The State Flight Academy of Ukraine, Kirovograd, 1996, 69 pp., Dep. to the State Scientific Library of Ukraine, October 24, 1996, № 2125–Uk96.
6. L.P. Petrenjuk, A.J. Petrenjuk, The existence of certain types of cubic decompositions of the graph K_{10} (Ukrainian). The Materials of the First Inter-High-Schools Seminar "Combinatorial Configurations and Their Applications", Kirovograd, April 19–20, 2006, 39–40.
7. L.P. Petrenjuk, A.J. Petrenjuk, The existence of cubic decompositions of order 10 (type 1102), The Materials of the Ninth International Science and Practice Conference "Science and Education – '2006", vol. 13, Physics and Mathematics. Dnipropetrovs'k, Science and Education, 2006, 64–68.
8. 8. L.P. Petrenjuk, A.J. Petrenjuk, On the existence of cubic decompositions of the graph K_{10} with the type 2011 (Ukrainian). Scientific Notes, vyp.65, Mathematical Sciences. Kirovograd State Pedagogical University, 2006, 85–94.
9. D.A. Petrenjuk, The census of possible types of cubic decompositions of K_{13} (Ukrainian). The Third International Science and Practical Conference 'Mathematical and Programming Maintenance of Intellectual Systems (MPZIS–2005)'. November 16–18, 2005, Report Theses, Dnipropetrovs'k, 2005, 137–138.
10. D.A. Petrenjuk. On the cubic decompositions of the graph K_{13}**.** The Materials of the First Inter-High-Schools Seminar "Combinatorial Configurations and Their Applications". Kirovograd, April 19–20, 2006, 41–42.