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SIMULATION OF SURFACE AND INTERNAL WAVE-PACKETS AND WIND WAVES IN TWO-LAYERED FLUID

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Проблема поширення внутрішніх і поверхневих хвиль у гідродинамічній системі "шар з жорстким дном – шар з вільною поверхнею" з урахуванням вітрових хвиль досліджена методом чисельного моделювання.

The problem of propagation of internal and surface waves in the hydrodynamic system "layer with rigid bottom – layer with free surface" taking into account wind waves is considered by the method of numerical simulation.

Introduction. The studying of wave processes in different fluid systems is one of the most actual problems of hydrodynamics. And in the recent moment, more and more numerical simulations and experimental researches are attached to a large number of analytical theoretical works.

Experimental studying of the interaction of internal and surface waves and field observations of these processes involves various optical methods and radar analysis, using special equipment that is installed on airplanes and satellites [4, 5].

Methods and approaches to study the dynamics of internal gravity waves are presented in [1]. These techniques combine simplicity and relative processing power expected to provide analytical results and conduct their qualitative analysis.

Studying of generation of internal waves on the boundary of the continental shelf and their research using satellite observations are carried out in [6]. Deviations of the ocean surface were interpreted as surface spots that occur during the propagation of internal wave packets.

Article [7] is devoted to full-scale observation of solitary internal waves with large amplitude in the South China Sea. The link between tidal currents and the emergence and spread of gravitational waves has been experimentally confirmed.

Using of a special spectrometer to record MODIS, internal waves in the South China Sea, in the Atlantic Ocean between Brazil and West Africa, in the equatorial Indian Ocean, in the eastern equatorial Pacific are described in [4].

An experimental technology using of microwave sensors for studying the internal wave packets is described in [5]. It allowed neutralizing the impact of weather conditions on observation.

In this article, the problem of propagation of internal and surface waves taking into account wind waves in the two-layered hydrodynamic system with rigid bottom and free surface is considered by the method of numerical simulation.

1. Statement of the problem. The mathematical statement of the problem for wave-packet propagation along the interface between the upper layer with free surface and a lower layer with rigid bottom is presented in the form

$$
\begin{aligned}
&\nabla^2\varphi_j = 0 \ \text{in} \ \Omega_j,\\
&\eta_{,t} - \varphi_{j,z} = -\alpha\varphi_{j,x}\eta_{,x} \ \text{at} \ z = \alpha\eta(x,t),\\
&\eta_{0,t} - \varphi_{2,z} = -\alpha\varphi_{2,x}\eta_{0,x} \ \text{at} \ z = \alpha\eta_0(x,t), \qquad (1)\\
&\varphi_{1,t} - \rho\varphi_{2,t} + (1-\rho)\eta + 0.5\alpha\left[(\nabla\varphi_1)^2 - \rho(\nabla\varphi_2)^2\right] - T\left(1+\alpha^2\eta_{,x}^2\right)^{-3/2}\eta_{,xx} = 0\\
&\qquad \text{at} \ z = \alpha\eta(x,t),\\
&\varphi_{2,t} + \eta_0 + 0.5\alpha(\nabla\varphi_2)^2 - T_0\left(1+\alpha^2\eta_{0,x}^2\right)^{-3/2}\eta_{0,xx} = 0 \quad \text{at} \quad z = \alpha\eta_0(x,t),\\
&\varphi_{1,z} = 0 \ \text{at} \ z = -h_1,
\end{aligned}
$$

where φ_j ($j=1,2$) are the velocity potentials; η and η_0 are the elevations of the interface and the free surface; $\rho = \rho_2/\rho_1$ is ratio of fluids densities; $\alpha = a/L$ is the nonlinearity coefficient; the lower fluid layer $\Omega_1 = \{(x,z): |x| < \infty, -h_1 \le z < 0\}$ and the upper fluid layer of $\Omega_2 = \{(x,z): |x| < \infty, 0 \le z \le h_2\}$. Dimensionless values were introduced using the characteristic length L, the maximal free surface elevation a, density of the lower fluid ρ_1, the acceleration of the gravity g.

2. Analytical solutions and numerical simulation. The solutions of the nonlinear problem (1) are determined using the method of multiple scale expansions up to the third-order approximation [2]. In [2, 3] for this problem is received dispersion equation

$$\omega^2 \operatorname{cth}(kh_1) + \rho\omega^2 \left(\frac{\omega^2 - (k + T_0 k^3)\operatorname{cth}(kh_2)}{\omega^2 \operatorname{cth}(kh_2) - (k + T_0 k^3)} \right) = (1-\rho)k + Tk^3 \qquad (2)$$

The dispersion equation (2) has two pairs of roots $\pm\omega_{1,3}$ and $\pm\omega_{2,4}$ corresponds to the frequencies of internal and surface waves. For each pair of roots can be written by independent solutions to the problem (1)

for ω_1

$$\eta_1^{(1)} = A\sin(kx - \omega_1 t),\ \eta_{01}^{(1)} = \frac{\omega_1^2}{\omega_1^2 \operatorname{ch}(kh_2) - (k + T_0 k^3)\operatorname{sh}(kh_2)} A\sin(kx - \omega_1 t), \qquad (3)$$

for ω_2

$$\eta_1^{(2)} = \frac{\omega_2^2 ch(kh_2) - (k + T_0 k^3) sh(kh_2)}{\omega_2^2} B\sin(kx - \omega_2 t),\ \eta_{01}^{(2)} = B\sin(kx - \omega_2 t), \qquad (4)$$

where A is amplitude of internal progressive wave $\eta_1^{(1)}$, B is amplitude of surface progressive wave $\eta_{01}^{(2)}$. The deviation of the contact surface and the free surface are given as follows

$$\eta_1 = \eta_1^{(1)} + \eta_1^{(2)}, \qquad \eta_{01} = \eta_{01}^{(1)} + \eta_{01}^{(2)}. \qquad (5)$$

Let's consider the wave-packets are consisting of three different waves with close lengths values

$$\eta = \sum_{i=1}^{3} \eta_1(k_i), \qquad \eta_0 = \sum_{i=1}^{3} \eta_{01}(k_i),$$

where η_1 and η_{01} determined by (5), $k_1 < k_2 < k_3$.

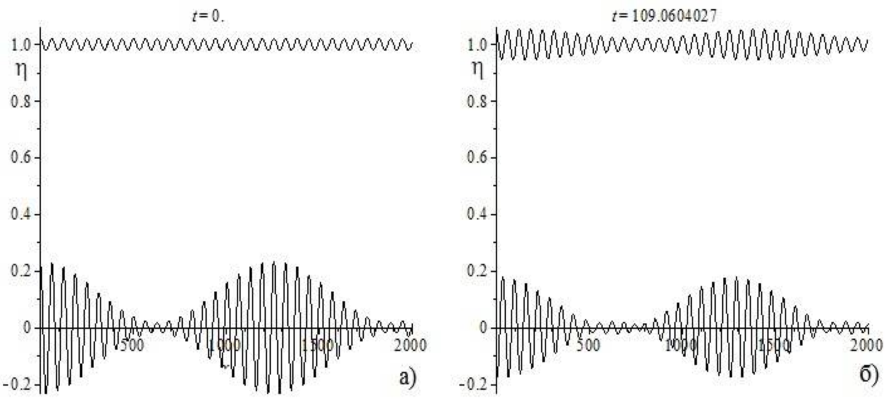

Fig. 1. Internal and surface wave-packets at time moments:

а) $t = 0$; б) $t = 109$

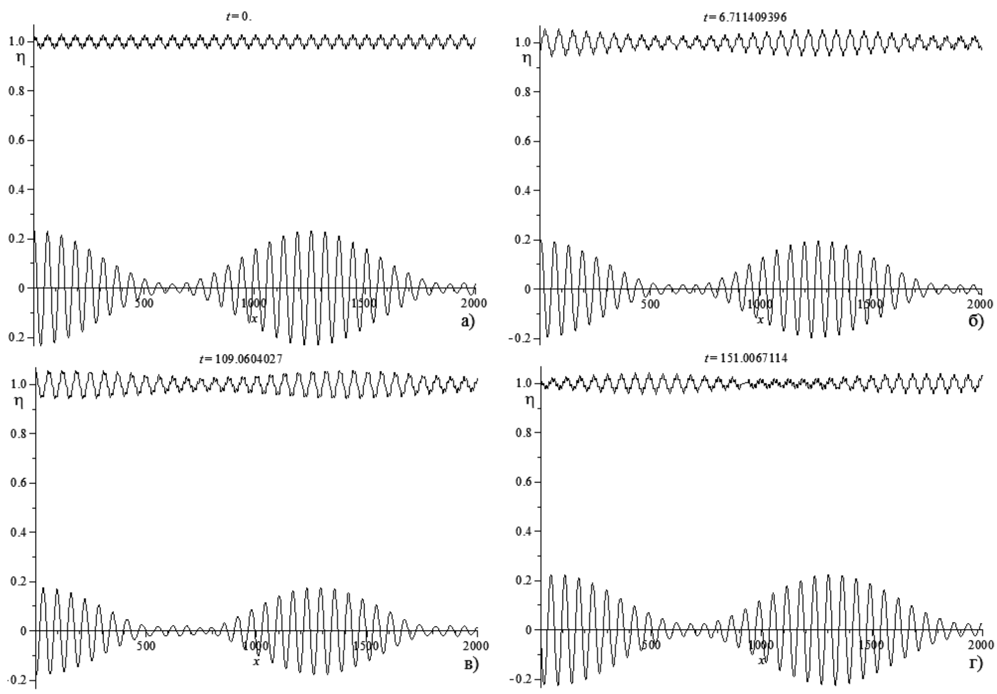

Fig.2. Internal and surface wave-packets with considering wind waves at time moments:
а) $t=0$; б) $t=6$; в) $t=109$; г) $t=151$

On the Fig.1, wave-packets are presented at different time moments for the following parameters of the system $A_1=0.1$, $A_2^0=0.03$, $h_1=10$, $h_2=1$, $\rho=0.9$, $T=0.0001$, $T_0=0.01$, $k_1=0.095$, $k_2=0.1$, $k_3=0.105$.

As it is shown on Fig.2, the availability of wind waves leads to the smoothing boundaries between groups of waves on the contact surface. The model makes it possible to establish qualitative patterns of surface and internal wave propagations.

The analysis of analytical solutions for different geometrical and physical parameters of the system is obtained in [3]. The occurrence of waves responses on the contact surface and the free surface were taken into account. The emergence of such waves takes place in the real conditions and it was confirmed by several observations. Analytical results allow carrying out a qualitative analysis of the motion of internal waves on the basis of the study of surface waves.

Conclusions. The analysis of the problem of propagation of internal and surface waves taking into account wind waves in the hydrodynamic system "layer with rigid bottom - layer with free surface" by the method of numerical simulation experiment was carried out. It was obtained that the availability of wind waves leads to the smoothing of boundaries between groups of waves on the contact surface.

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