

## ADVANTAGES AND DISADVANTAGES OF USING MATHEMATICAL SOFTWARE IN THE WORK OF MATHEMATICAL CIRCLE

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We consider a number of competitive trigonometry problems, the condition of which is to build a graph of a function or equation. These problems are solved with the using of the popular software. In the process of work of these applications, one can find some advantages and disadvantages.

Розглядається низка конкурсних задач тригонометрії, умовою яких є побудова графіка функції чи рівняння. Задачі розв'язуються з використанням популярних комп'ютерних програм, в процесі роботи з якими встановлюються недоліки і знаходяться переваги їх використання.

The mathematical circle is a form of extracurricular activities in mathematics in high school. It is very important today that pupils can extend and deepen their knowledge, learn to solve math problems, enhance their mathematical training in the lessons of mathematical circle. Mathematics is becoming a meaningful and a required subject, because more and more professions require a high level of application of mathematics.

Sometimes teachers need to demonstrate accurate graphing of functions with a clear display of all steps of constructing. It should be to use existing software and possibilities of ICT. A lot of scientific works were devoted to use of these tools in the study of mathematical disciplines. In particular, research papers of M.Zhaldak, S.Rakov, S.Semerikov, T.Kramarenko, Y.Trius, E.Vinnichenko, I.Lupan, O.Spivakovsky and other authors allow to investigate this problem thoroughly. There are cases when mathematical programs do not work correctly. This is due to the functions embedded in them. This article written about the advantages and disadvantages of using ICT in the teaching of mathematics, particularly during the lessons of mathematical circle.

One of the topics that would be useful to consider with the senior pupils is "Competitive trigonometry problems" [1]. There are considered the problems of graphing of functions which containing trigonometric functions, a module, an integer part and a fractional part, inverse trigonometric functions etc.

*Example 1.* Construct a graph of the function:  $y = [-3 \sin x]$ . (1)

*Solving:* Assume that we must solve this problem hand-operated. Therefore, we do sequential transformation with the graphs of functions:

1)  $y = \sin x$  – sinusoid; 2)  $y = \sin 2x$  – compress 1) twice to the vertical axis; 3)  $y = 3 \sin 2x$  – stretch 2) three times on the horizontal axis; 4)  $y = -3 \sin 2x$  – expose 3) symmetric in relation to the horizontal axis; 5)  $y = [-3 \sin 2x]$  – use the algorithm of constructing the graph of integer part.

The algorithm of constructing the graph  $y = [f(x)]$ : to construct a graph of function  $y = [f(x)]$ , we must build straight lines  $y = k, k \in Z$ , and the parts of the graphics of function  $y = f(x)$ , those contained in the band  $k \leq y < k + 1$ , we will project for lower limit  $y = k$  of the band; a set of constructed projections is a sought-for graph [2].

As you can see you have to make difficult accurate transformations in order to get correct, understandable and visual picture. So we advise to use The Advanced Grapher, which is one of the most convenient program for such purpose. As a result of using this program, we received this picture (figure 1):

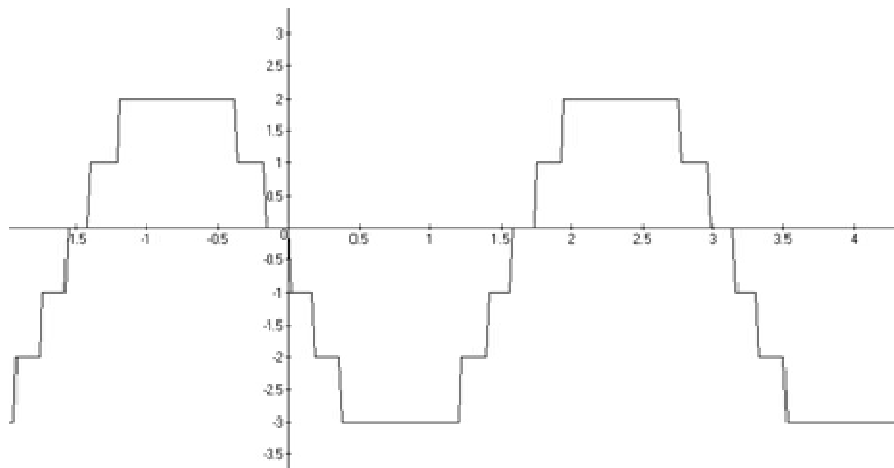


Figure 1. Image the graphic of function  $y = [-3 \sin 2x]$  in the Advanced Grapher

Considering the range of values sinus, we can say that the minimum value of a given function (1) is -3, and the maximum value is 3. But you can see that in figure 1 no maximum points. If you use a powerful mathematical package Maple 15 to build the same graphic, the picture will be as follows (Figure 2):

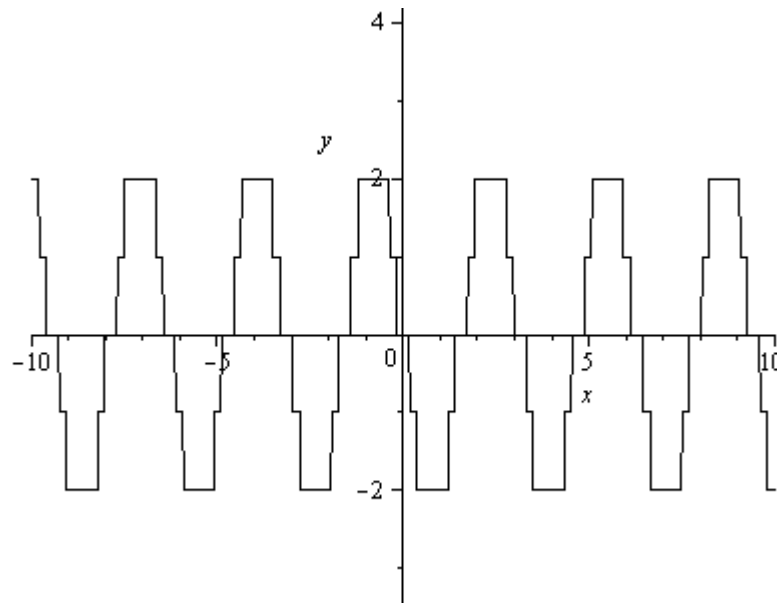


Figure 2. Image the graphic of function  $y = [-3 \sin 2x]$  in the Maple 15

Figure 2 also is not true. This example demonstrates that we should not blindly trust to the mathematical programs without a thorough preliminary analysis of the problem.

Example 2. Construct a graph of the function: 
$$y = \frac{|\cos 3x|}{\cos 3x} \cdot x^2 \tag{2}$$

Solving: 1). At first we find the domain of definition this function:

$$\cos 3x \neq 0 \Rightarrow 3x \neq \frac{\pi}{2} + \pi n, n \in Z \Rightarrow x \neq \frac{\pi}{6} + \frac{\pi n}{3}, n \in Z \tag{3}$$

2). Then we open the module at the intervals, where expression  $\cos 3x$  keeps its

$$\text{sign: } y = \begin{cases} \frac{\cos 3x}{\cos 3x} x^2, \cos 3x > 0 \\ -\frac{\cos 3x}{\cos 3x} x^2, \cos 3x < 0 \end{cases} \Rightarrow y = \begin{cases} x^2, \cos 3x > 0 \\ -x^2, \cos 3x < 0 \end{cases} \tag{4}$$

Construct a graph of the function, taking into account the conditions (3) and a piecewise setting function (4). We use a teaching program complex "Live Mathematics" and Advanced Grapher for plotting function (2). Maple program displays the exact same graph as the program Advanced Grapher. Preliminary analysis of function shows that the graph should have break points type jump. The pupils should see these break points in received graphs. But on the right in Figure 3 you can see a continuous graph and on the left in Figure 3 the graph is continuous

only at an interval  $\left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$ . Although the points  $\pm \frac{\pi}{6}$  belonging to the interval  $\left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$  should be break points.

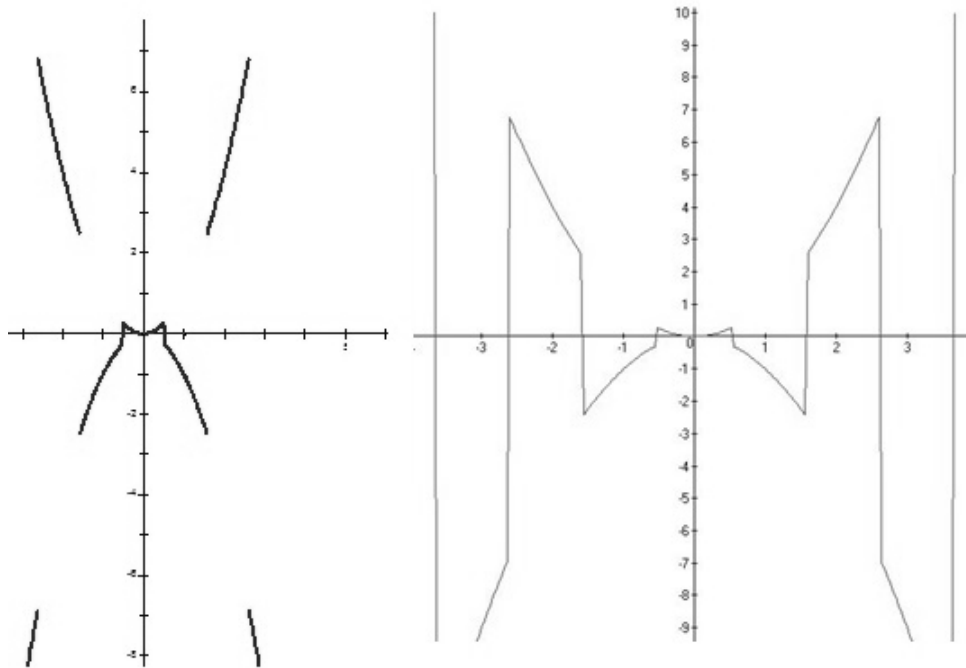


Figure 3. Images the graphic of function  $y = \frac{|\cos 3x|}{\cos 3x} \cdot x^2$  in the "Live Mathematics" (left), the program Advanced Grapher (right)

This example again points to the flaws that arise when we use ICT in the classroom with the pupils. But you can't argue that the use of computer programs while studying mathematics inappropriate.

In particular, ICT have some advantages. For example, the limitation of blackboard does not allow to build the pieces of graph, that are above or below a certain point of the vertical axis (pupils must imagine these pieces of graph). Computer programs allow display the parts of graphics (2) that are lying arbitrarily far from the origin. In accordance, it contributes to the development of spatial imagination of the pupils.

Example 3. On the coordinate plane OXY build a geometric image of equation

$$\operatorname{tg}(\pi(|x| + |y|)) = 0 \tag{5}$$

*Solving:* We solve this trigonometric equation  $\operatorname{tg}(\pi(|x| + |y|)) = 0 \Rightarrow \pi(|x| + |y|) = \pi k, k \in N \Rightarrow |x| + |y| = k, k \in N$ . The last equality is a set of squares with the "integer" tops on the axes.

It is not difficult to construct the resulting geometric image of given equation by hand. But still we want to use a computer and get a good illustration of the solution. Unfortunately, not all math programs have built-in functions to directly graphing of equations. Advanced Grapher program allows to do it and you get this picture (Figure 4):

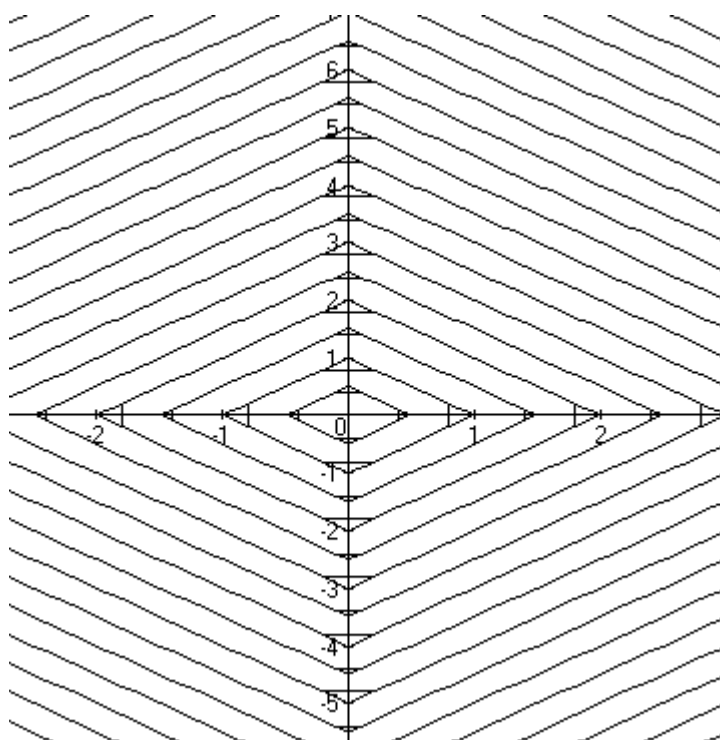


Figure 4. The geometric image of equation  $\operatorname{tg}(\pi(|x| + |y|)) = 0$  received in the program *Advanced Grapher*

**Example 4.** Construct a graph of the function:  $y = \cos(2 \arcsin x)$ . (6)

*Solving:* 1) We find the domain of definition this function:  $D(y) = [-1; 1]$ ;

2) after we denote  $\arcsin x = \alpha$ , then  $x = \sin \alpha$  and  $-\frac{\pi}{2} \leq \alpha \leq \frac{\pi}{2}$ ; 3) thus, the function

(6) will be written in the form  $y = \cos 2\alpha = 1 - 2\sin^2 \alpha = 1 - 2x^2$ ; 4) therefore, the graph of function (6) is an arc of a parabola  $y = 1 - 2x^2$  when  $x \in [-1; 1]$ .

The analysis and the resulting solution of the problem allows us to construct a graph of given function (6) by hand. But try to test our software by making each of

them request «plot output function (6)». You can see the results obtained in the programs on the following Figure 5.

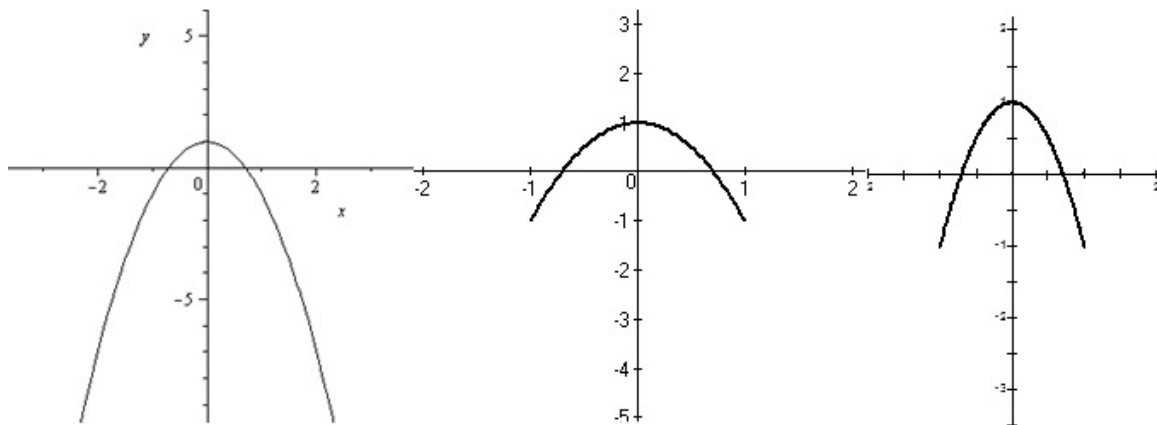


Figure 5. Image the graphic of function  $y = \cos(2 \arcsin x)$  received in the programs: Maple, Advanced Grapher, «Live Mathematics» (respectively from left to right)

Obviously, the last two programs give the correct result - an arc of a parabola  $y = 1 - 2x^2$  on the interval  $x \in [-1;1]$ , but Maple program built parabola completely on the set of all real numbers instead of a piece of it.

Example 5. Construct a graph of the function:  $y = \arccos(\cos x)$ . (7)

*Solving:* 1) The function is periodic with the period  $2\pi$  ( its value is not changed when  $x$  change to  $x + 2\pi$  ). So enough build graph in the interval  $0 \leq x \leq 2\pi$  , and then move it along horizontal axis several times left and right; 2) function (7) is converted to  $y = x$  when  $0 \leq x \leq \pi$  (this follows from the definition of arc cosine); 3) considering that  $\cos x = \cos(2\pi - x)$  and when  $\pi \leq x \leq 2\pi$  then inequality  $0 \leq 2\pi - x \leq \pi$  is true. Then by definition the arc cosine we get that  $y = 2\pi - x$ .

Graph of function (7) is not even hard to build by hand, but we will continue testing our programs. For this problem all programs and Maple, and «Live Mathematics», and Advanced Grapher display the identical correct results, so we only give image obtained in the program Maple (Figure 6).

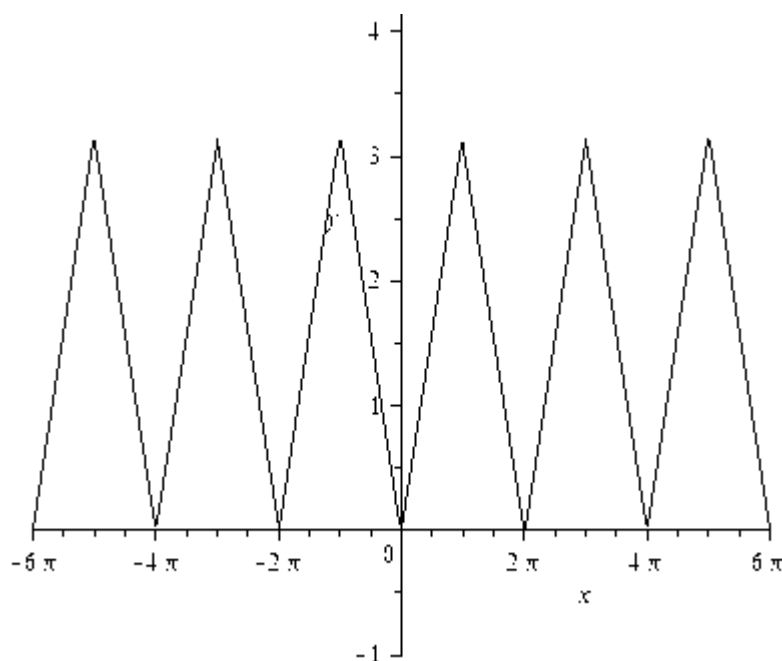


Figure 6. Image the graphic of function  $y = \arccos(\cos x)$  in the Maple

These competitive mathematical problems of trigonometry in which you must to construct the graphs of functions or equations can be used in the lessons of mathematical circle or extracurricular activities with the senior pupils. In this article, we analyzed the use of ICT in the learning of solving problems of this type. The analysis shows that mathematical computer programs do not always give the correct result, therefore they not only simplify and improve the learning process, and vice versa. Consequently, you must always do depth analysis of conditions and the possible results in solving problems with the using the ICT, otherwise you will not avoid mistakes and incorrect mathematical concepts in the pupils. But in most cases, mathematical software allow to reduce the time that would be spent on the complete problem solving, stimulate the development of spatial thinking of the pupils, form a strong interest in mathematics and computer science.

This article may be useful to teachers and students of faculty of Physics and Mathematics of the Pedagogical Universities.

#### REFERENCES

- [1] Gorodnichenko V. Competitive trigonometry problems// Mathematics in schools of Ukraine. – №1-2 (301-302), 2011. – P.34-37.
- [2] Voronyy A. Selected problems of school mathematics: teach. manual / A. Voronyy – Kirovograd: [not publishing], 2004. – 231 p. – (Preparing for the Olympiad in mathematics).