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TWO STOCHASTIC MODELS FOR AIR TRAFFIC CONTROL OPTIMIZATION

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Розглянуто дві стохастичні оптимізаційні моделі: для системи планування польотів вантажних літаків, що обслуговує рейси двох типів, та для системи планування польотів регулярних та спеціальних пасажирських рейсів.

Рассмотрены две стохастические оптимизационные модели: для системы планирования полетов грузовых самолетов, обслуживающих рейсы двух типов, и для системы планирования полетов регулярных и специальных пассажирских рейсов.

Introduction. Stochastic programming is a science that is developing rapidly and covers many areas of economic activity. Stochastic programming models use the knowledge of probability distributions for data or estimates and provide stochastic definition of the objective function. In the practical tasks of management and planning at incomplete information a decision should be taken quickly to monitor the implementation of random input data but further accumulation of information on preliminary plan can be corrected.

Due to the increased of use of air traffic an air transport traffic are of particular importance in the development of the economy. Now rather important is to create conditions for the safe, quality and efficient air navigation services and aircraft modernization analysis system for quality surveillance in air traffic control centers.

Stochastic programming defines a new approach to management algorithms in complex systems. This is expedient to combine of complex mathematical support of extremal control systems not of algorithms for solving extreme problems but with decision rules from relevant stochastic extensions. Formation of control laws are not associated with the operational work, but on the stage of designing of the control system.

Review of preliminary results is held to consider air traffic control system (here are some of them: Ermolev Yu.M. [3], Zhurbenko N.G. [4], Yudin D.B. [6], Birge J. R. & Louveaux F. [7]), and methods of solving of stochastic programming problem (Wagner G. [1], Holenko D.I. [2], Yudin D.B. [5], Shapiro A, Dentcheva D, Ruszczyński A. [8], Stein W. Wallace & William T. Ziemba [9]).

This article discusses two problems of modeling air traffic control of cargo and passenger aircraft on condition of incomplete information

1. Problem of optimization of flight planning cargo aircraft of several different types. Let's consider the flight planning system of cargo aircraft serving flights of two types (regular and special). The regular flights are conducted between fixed points and planned for each month in advance. The special flights occur regularly, time and freight items are not fixed in advance.

The problem is formulated as a two-step stochastic one. In the first stage, before the application will be announced on a special flight, the aircraft of each type are distributed between routes and number of flights of aircraft of each type on each line is determined [6]. Reassignment of aircraft from one route to another is performed on the second stage after the establishment of the implementation of random parameters of conditions. We introduce the following notation:

x_{ij} is the number of flights during the month of aircraft type i, assigned to the route j;

x_{ijk} is the number of flights taken from the aircraft route j and override to the route k;

y_j^+ is the dissatisfied proposal (in tonnes of cargo) of traffic on route j;

y_j^- is not loaded aircraft capacity (tonnes of cargo) on route j;

a_{ij} is the number of hours required to aircraft type i to overcome route j, if the plane was originally scheduled for this route;

a_{ijk} is the number of hours required aircraft type i, initially designated to the route j, in order to overcome route k;

b_{ij} is the number of tons of cargo carried per flight by plane type i on the route j;

a_i is the number of flight hours permissible per month of the aircraft type i;

d_j is the application for traffic (tonnes of cargo) on the route j;

c_{ij} is the cost of aircraft flight type i on the route j, provided that the aircraft was originally designed on this route;

c_{ijk} is the cost of aircraft flight type i on the route k, if it was removed from the route j;

q_j^+ is the penalty for dissatisfaction application for traffic per ton of cargo on the route j;

q_j^- is the penalty for underemployment per ton aircraft on the route j.

The model of the problem can be written in the form.

The objective function $\sum_{i,j} c_{ij}x_{ij} \to min$.

Restrictions for the first stage limit for each type of aircraft the total number of flight hours on all routes

$\sum_j a_{ij}x_{ij} \le a_i, \forall i,\quad \sum_j a_{ij}x_{ij} \ge 1,\ \forall i, \sum_j x_{ij} \ge 5 \forall i, x_{ij} \ge 0, x_{ij}$ – are integer values.

The objective function for the second stage

$$\sum_{i,j}\sum_{k\ne j}(c_{ijk} - c_{ij} * (a_{ijk}/a_{ij})x_{ijk} + \sum_j(q_j^+ y_j^+ + q_j^- y_j^-) \to min.$$

Restrictions for the second stage

$\sum_{k\ne j}(a_{ijk}/a_{ij})x_{ijk} \le x_{ij}, \forall i,j;$

$\sum_i b_{ij}x_{ij} + \sum_i\sum_{k\ne j} b_{ij}x_{ijk} - \sum_i\sum_{k\ne j}(b_{ij} * (a_{ijk}/a_{ij})) * x_{ijk} + y_j^+ - y_j^- = d_j, \forall j;$

$\sum x_{ijk} \ge z_i, z_i = 1; y_j^+, y_j^- \ge 0; x_{ij} \ge x_{ijk}; x_{ijk} \ge 0, x_{ijk}$ – are integer values.

Let’s consider the problem with the following parameters:

$$a_{ij} = \begin{pmatrix} 5,1 & 2 & 3 \\ 1 & 4 & 2 \\ 2,3 & 5 & 3,2 \end{pmatrix};$$

$$a_{1jk} = \begin{pmatrix} 6 & 6,2 & 5,3 \\ 5,4 & 7 & 5,8 \\ 6,7 & 5,5 & 6,1 \end{pmatrix}, a_{2jk} = \begin{pmatrix} 7,1 & 6,3 & 5,1 \\ 5,3 & 5,7 & 5,6 \\ 6,2 & 7,3 & 6 \end{pmatrix}, a_{3jk} = \begin{pmatrix} 5,5 & 5,1 & 6 \\ 6,2 & 5,4 & 7 \\ 7,7 & 6,7 & 5,9 \end{pmatrix};$$

$$b_{ij} = \begin{pmatrix} 8 & 7 & 7 \\ 8,9 & 11 & 10,4 \\ 13,1 & 9,2 & 12 \end{pmatrix};$$

$$a_i = \begin{pmatrix} 80 \\ 90 \\ 70 \end{pmatrix}, d_j = \begin{pmatrix} 35,5 \\ 41 \\ 52,1 \end{pmatrix}, c_{ij} = \begin{pmatrix} 1,2 & 2 & 2,3 \\ 3 & 1,4 & 1 \\ 1 & 1,5 & 2,1 \end{pmatrix};$$

$$c_{1jk} = \begin{pmatrix} 5,7 & 10 & 8,2 \\ 8 & 6,4 & 6,6 \\ 7,8 & 9 & 5,8 \end{pmatrix}, c_{2jk} = \begin{pmatrix} 11 & 9,4 & 7,2 \\ 8,1 & 7,5 & 8,8 \\ 6,5 & 10 & 9,7 \end{pmatrix}, c_{3jk} = \begin{pmatrix} 7 & 9,1 & 6,2 \\ 8 & 10,4 & 9 \\ 6,8 & 8,9 & 7,3 \end{pmatrix};$$

$q_j^+ = (5 \quad 6 \quad 5),\ q_j^- = (2 \quad 3 \quad 4).$

We expect non-negative parameters x_{ij}, x_{ijk}, y_j^+, y_j^-. The problem was solves used “Search solutions” from MSExcel, which works with a group of cells, directly or indirectly related to the formula in the target cell and allows to find the optimal value for the target function. Solution of one-step stochastic programming problem is presented in Figure 1.

(a)

x_ij	1	1	0
	0	0	1
	2	0	0

(b)

x_ijk	1	0	0	0	0	0	0	1	0
	0	0	0	0	0	0	0	0	0
	0	0	0	1	0	0	0	0	0

(c)

y_j	1,3	34	41,7	0	0	0

Fig.1. Solution in MS Excel environment: (a) x_{ij}; (b) x_{ijk}; (c) y_j^+, y_j^-.

That is the type of routes x_{11} and x_{12} one flight is removed and reassigned to routes x_{111} and x_{312}, respectively, and the type of route x_{31} flight is reassigned to the route x_{231}.. The value of dissatisfied requests on the route *j* in tonnes of cargo reaches 1.3; 34 and 41.7 tonnes on each flight, respectively. The costs of data flight are equal to 469 conventional units.

2. Problem of optimization of flights of passenger aircrafts. Let's consider two types of passenger flights: regular and special. The demand for special traffic is unknown. Number of seats that goes during the day cannot be provided in full. After some time Information about the parameters of the conditions of the problem comes. There is a need for reassignment of aircraft routes, which received fewer applications than expected on the line serving the transportation, for which demand was higher than expected one. For setting the problem it was selected following types of aircraft that will carry out flights: Airbus A320, Boeing 737-700, MD-81. Maximum passenger capacity is respectively 179, 149 and 172 the number of people. The maximum flight duration: 7.3; 7.4 and 3.6 hours, respectively. Desired unknown values are non-negative values: x_{ij}- the number of flights during the month of aircraft type i, assigned to the route j; x_{ijk}- the number of flights taken from the aircraft route j and override to the route k; y_j^+- dissatisfied proposals (in number of people) for the transportation on the route j; y_j^- - number of empty seats on the route j. For a complete research of the problem additional restrictions and requirements for the construction of models including partial case of insufficient number of passengers should be considered. So, conditions of maximum filling of seats in aircraft of the selected type to be additionally considered for minimizing the traffic cost.

Conclusions. Stochastic optimization model for the system of cargo aircraft flight planning, which serves two types of flights is considered; stochastic model of the problem of selection parameters is studied; the basic parameters for the formulation of the problem is identified; restrictions that allow to find the optimal solution plan is set; stochastic solution of specific problem is received; it was found that in terms of the problem with all the limitations the optimum reassigning are flights once reassignment to another route, and the costs will be minimal.

Formulation of optimization problem of stochastic programming about special regular passenger flights is formulated and the method of its solution is offered.

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