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SUPERADDITIVITY AND VECTORS MEASURES IN BIOLOGICAL SYSTEMS

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It is shown that the resulting functional of process of biosynthesis has the property of superadditivity.

Показано, що результуючий функціонал процесу біосинтезу має властивість суперадитивності.

Introduction. Processes of biosynthesis are the significant factors for the development of biological systems. Therefore it is important to investigate such processes by mathematical methods.

The statement of problem. Let S be the capillary net which delivers the amino acids. Let us denote by p_k the masses of amino acids that it necessary for the synthesis of 1g of protein, where k = 1,…,20.

Distribution of each amino acid is defined by some scalar measure m_k with nonnegative values. The vector measure with scalar components m_k (k = 1,…,20) we denote by m.

Definition1. The vector p = (p_1,…,p_{20}) is called *the parameter vector of process of biosynthesis.*

Definition 2. The functional $f(y) = minimum(y_k/p_k$, k=1,…,20) is called *the resulting functional of biosynthesis.*

Definition 3. The process of biosynthesis is called *optimal*, if after the finish this process unused acid dos not remain.

The **aim** of present paper is investigation of properties of resulting functional of biosynthesis.

The results of paper.

Theorem 1. *Suppose that the process of biosynthesis in capillary net S is defined by vector measure m and has the parameter vector p. Then this process is optimal if and only if when the vectors p and m(S) are collinear.*

Theorem 2. *Under the assumptions of Theorem 1 the mass of synthesized protein is equal to the value of the resulting functional on the vector m(S).*

The proofs of Theorems 1 and 2 are not difficult.

Recall that a functional g is called *superadditive* if for all vectors x and y there is true the inequality

$$g(x+y) \geq g(x)+g(y).$$

Theorem 3. *The resulting functional of biosynthesis has the property of superadditivity.*

Proof. Let us consider any two vectors x and y. For each number k it is true the inequality

$$(x_k+y_k)/p_k) \geq minimum(x_j/p_j, j = 1, \dots,20) + minimum(\frac{y_j}{p_j}, j = 1,\dots,20) =$$

f(x)+f(y).

Let us go to minimum in the left part of this inequality. Then we obtain that it is true the inequality $f(x+y) \geq f(x)+f(y)$. The theorem is proved.

Remark 1. The results of Theorems 1, 2 and 3 are true for the ecological systems.

Remark 2. There are more than 2 millions biological species in the Earth. Therefore there exist the ecological systems with very numerous components.

Remark 3. For the systems with very big quantity of components the effect of superadditivity is more essential.

Remark 4. Because of structure of resulting functional, the best topology in the space of values of vector measures is the topology which is generated by uniform metric. For the big quantity of coordinates the multidimensional space with uniform metric is almost analogous to the space of bounded sequences. It is known that for the space of bounded sequences the different definitions of continuous translations and different definitions of differentiability directions are nonequivalent [1], [2]. Therefore for the systems with big quantity of components it is necessary to investigate the dynamic of process by detail investigation of increments of corresponding vector measures.

REFERENCES

[1] Romanov V.A. On the nonequivalence of three definitions of continuous translations for vector measures. Mathematical Notes. – 1995. – Volume 57, No. 1-2. – P. 220-222.
[2] Romanov V.A. Nonequivalence of various definitions of differentiability directions for vector measures. Ibid. – 2002. – Volume 72, No. 4. – P. 489-494.